

Robotics 101:Part 1:

4. a)  $\cos a = \frac{x_1}{L_1}$   
 b)  $x_1 = L_1 \cos a$   
 c)  $x_2 - x_1 = L_2 \cos(a+B)$   
 (distance between  $x_2$  and  $x_1$ )  
 d)  $x_2 = L_2 \cos(a+B) + L_1 \cos a \rightarrow$  "Equation 1"
5.  $y_2 = L_2 \sin(a+B) + L_1 \sin a \rightarrow$  "Equation 2"

Part 2:

6.  $x_2 = 6$      $y_2 = 2$   
 $L_1 = 5$      $L_2 = 3$   
 $a = ?$      $B = ?$
- a)  $6 = 3 \cos(a+B) + 5 \cos a$   
 $2 = 3 \sin(a+B) + 5 \sin a$
- b)  $(3 \cos(a+B) + 5 \cos a)^2 + (3 \sin(a+B) + 5 \sin a)^2 = 40$
- c)  $[(3 \cos(a+B) + 5 \cos a)(3 \cos(a+B) + 5 \cos a) + (3 \sin(a+B) + 5 \sin a)(3 \sin(a+B) + 5 \sin a)]$   
 \* See Equations Sheet

d)  $\frac{1}{5} = \cos B$

e)  $B = \cos^{-1}\left(\frac{1}{5}\right) = 78.46^\circ$

d)  $\cos B = \frac{1}{5} = \frac{5}{25}$

$5^2 - 1^2 = 24 \rightarrow \sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$

$\sin B = \frac{2\sqrt{6}}{5}$

$B = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right) = 78.46^\circ$

Part 3:

7. a)  $x_2 = L_2 \cos a \cos B - L_2 \sin a \sin B + L_1 \cos a$   
 $x_2 = \cos a (L_2 \cos B + L_1) - L_2 \sin a \sin B \rightarrow$  "Equation 4"  
 $y_2 = L_2 \sin a \cos B + L_2 \cos a \sin B + L_1 \sin a$   
 $y_2 = \sin a (L_2 \cos B + L_1) + L_2 \cos a \sin B \rightarrow$  "Equation 5"
- b)  $6 = \cos a (3 \cdot \frac{1}{5} + 5) - 3 \sin a \cdot \frac{2\sqrt{6}}{5}$   
 $6 = \frac{28}{5} \cos a - \frac{6\sqrt{6}}{5} \sin a \rightarrow$  "Equation 6"  
 $2 = \sin a (3 \cdot \frac{1}{5} + 5) + 3 \cos a \cdot \frac{2\sqrt{6}}{5}$   
 $2 = \frac{28}{5} \sin a + \frac{6\sqrt{6}}{5} \cos a \rightarrow$  "Equation 7"



$$6. b.) [(3\cos(a+B) + 5\cos a)(3\cos(a+B) + 5\cos a)] + [(3\sin(a+B) + 5\sin a)(3\sin(a+B) + 5\sin a)]$$

$$40 = 9\cos^2(a+B) + 30\cos(a+B)\cos a + 25\cos^2 a + 9\sin^2(a+B) + 30\sin(a+B)\sin a + 25\sin^2 a$$

$$40 = 34 + 30(\cos(a+B)\cos a + \sin(a+B)\sin a)$$

$$\frac{6}{30} = \cos(a+B)\cos a + \sin(a+B)\sin a$$

$$\frac{1}{5} = (\cos a \cos B + \sin a \sin B)\cos a + (\sin a \cos B + \cos a \sin B)\sin a \rightarrow \text{"Equation 3"}$$

$$\frac{1}{5} = \cos^2 a \cos B - \sin a \sin B \cos a + \sin^2 a \cos B + \cos a \sin B \sin a$$

$$\frac{1}{5} = \cos^2 a \cos B + \sin^2 a \cos B$$

$$\frac{1}{5} = \cos B$$

Sheet 1



7. c.) Let  $\cos a = x$  and  $\sin a = y$

$$6 = \frac{28}{5}x + \frac{6\sqrt{6}}{5}y$$

$$2 = \frac{6\sqrt{6}}{5}x + \frac{28}{5}y$$

$$D = \begin{vmatrix} \frac{28}{5} & -\frac{6\sqrt{6}}{5} \\ \frac{6\sqrt{6}}{5} & \frac{28}{5} \end{vmatrix} = 40$$

$$\begin{cases} \frac{28}{5}x - \frac{6\sqrt{6}}{5}y = 6 \\ \frac{6\sqrt{6}}{5}x - \frac{28}{5}y = 2 \end{cases}$$

$$x = \frac{\begin{vmatrix} 6 & -\frac{6\sqrt{6}}{5} \\ 2 & \frac{28}{5} \end{vmatrix}}{\begin{vmatrix} \frac{28}{5} & -\frac{6\sqrt{6}}{5} \\ \frac{6\sqrt{6}}{5} & \frac{28}{5} \end{vmatrix}} = \frac{D_x}{D} = \frac{39.48}{40} = .987$$

$$y = \frac{\begin{vmatrix} \frac{28}{5} & 6 \\ \frac{6\sqrt{6}}{5} & 2 \end{vmatrix}}{\begin{vmatrix} \frac{28}{5} & -\frac{6\sqrt{6}}{5} \\ \frac{6\sqrt{6}}{5} & \frac{28}{5} \end{vmatrix}} = \frac{D_y}{D} = \frac{-6.45}{40} = -.161$$

d)  $x = .987 \rightarrow a = \cos^{-1}(.987) = 9.25$

$y = -.16 \rightarrow a = \sin^{-1}(-.16) = -9.26$

Part IV:

1. a)  $x_1 = L_1 \cos a$

$$x_1 = 3 \cos 45 = 2.12$$

$$x_2 = L_2 \cos(a+\beta) + L_1 \cos a$$

$$x_2 = 5 \cos(75) + 2.12 = 3.41$$

$$y_1 = L_1 \sin a$$

$$y_1 = 3 \sin 45 = 2.12$$

$$y_2 = L_2 \sin(a+\beta) + L_1 \sin a$$

$$y_2 = 5 \sin(75) + 2.12 = 6.95$$

b)  $x_1 = 3 \cos(-45) = 2.12$

$$x_2 = 5 \cos(-15) + 2.12 = 6.95$$

$$y_1 = 3 \sin(-45) = -2.12$$

$$y_2 = 5 \sin(-15) - 2.12 = -3.41$$

c)  $x_1 = 3 \cos 45 = 2.12$

$$x_2 = 5 \cos 15 + 2.12 = 6.95$$

$$y_1 = 3 \sin 45 = 2.12$$

$$y_2 = 5 \sin 15 + 2.12 = 3.41$$

See graphs on separate sheet



Part IV:  
1. a-c)

